Cosmology after Phantom Crossing by Horndeski Gravity

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One possible way to explain the observed effective dark energy equation of state crossing $w = -1$ (the phantom divide) is through modified gravity. A key point is to not view the expansion history in isolation but to take into account the other gravitational impacts on growth of large scale structure, lensing, etc. Within shift symmetric Horndeski gravity this implies three main paths for the late time cosmic expansion. All require unusual kinetic structure and we analyze their various implications for how w should behave after phantom crossing.

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I. INTRODUCTION

Current cosmic data points toward a "beautifully bizarre" revolutionary picture of dark energy that is not only dynamic but evolves across the phantom divide, from effective equation of state ratio $w < -1$ at high redshift to $w > -1$ at late times [1–3]. Conventional (noninteracting, canonical, general relativistic) dark energy cannot achieve this [4–7].

Both interacting and modified gravity models designed to enable this behavior of the cosmic expansion tend to run afoul of cosmic growth (see [8] and references therein). That is, if they couple in some way to matter (either through direct interactions, e.g. [9, 10], or "phake phantoms" [7, 11, 12]) they will significantly alter the growth of large scale structure [8]. On the other hand if they couple nonminimally to gravity, modifying general relativity (e.g. [8, 13]), they again impact cosmic growth, and possibly also light propagation (lensing).

Here we consider modified gravity but seek to temper the impact on growth and lensing, as well as focusing on how the dark energy behaves at late times. That is, does the rapid rise of w after crossing -1 continue, level off, or return to $w = -1$?

In Section II we highlight the key aspect of considering cosmic expansion and gravitational effects, e.g. on growth of large scale structure and on lensing, simultaneously. Section III looks at the implications if gravitational braiding effects are large, and how dark energy then must cede its dominance. For the observed condition of dark energy dominance, Section IV explores the future behavior of the dark energy equation of state under "natural" conditions, while following sections consider deviations from naturalness (Section V: kinetic structure; Section VI: field evolution; Section VII: field coasting; Section VIII: vanishing kinetic term). We conclude in Section IX.

II. CONNECTING COSMIC EXPANSION AND GROWTH

Horndeski gravity provides an excellent framework for exploring modified gravity, being highly general and well behaved. Observations have constrained some of the generality, in particular removing the G_5 term in the action and restricting the G_4 term to being a function of the scalar field ϕ but not its derivative $X \equiv \dot\phi^2/2$ (in the most straightforward interpretation of gravitational waves propagating at the speed of light). Thus one has a Lagrangian

$$\mathcal{L} = G_4(\phi)\, R + K(\phi, X) - G_3(\phi, X)\, \Box\phi \ , \tag{1}$$

where K is the kinetic term.

Theory motivates restriction to shift symmetry in order to rein in quantum corrections. If one adopts this then one is left with

$$\mathcal{L} = \frac{1}{2}R + K(X) - G_3(X)\, \Box\phi \ , \tag{2}$$

normalizing the Planck mass to $M_{\rm Pl}^2 = 1$. Note that since $G_4(\phi)$ is what led to issues with cosmic growth as seen in [8, 13], the shift symmetry has an added benefit of easing the fit to observations.

From this action the equations of motion yield the modified Friedmann equations

$$3H^2 = \rho_m + \rho_{\rm de,eff} \ , \tag{3}$$

$$-2\dot H = \rho_m + P_m + \rho_{\rm de,eff} + P_{\rm de,eff} \ , \tag{4}$$

$$0 = \ddot\phi \left[K_X + 2XK_{XX} + 6H\dot\phi g_X\right] + 3H\dot\phi K_X + 6g\left(\dot H + 3H^2\right) \ . \tag{5}$$

Hereafter we will simply write dark energy rather than effective dark energy and suppress the eff subscript. We take the matter component to be pressureless, $P_m = 0$, and it follows the usual continuity equation, avoiding a "phake phantom" as well as various impacts on large scale structure growth.

The third equation in the set is the scalar field equation of motion. Here $g \equiv XG_{3X}$ and a subscript X denotes a derivative with respect to X. We can readily see that the equation reduces to the usual Klein-Gordon equation in general relativity, i.e. $K = X$, $g = 0$.

The explicit forms of the dark energy density and pressure are

$$\rho_{\rm de} = -K + 2XK_X + 6H\dot\phi g \ , \tag{6}$$

$$P_{\rm de} = K - 2g\ddot\phi \ . \tag{7}$$

term is negative. This does not automatically hold here. The g term also enters in the no ghost condition, $\alpha_K+(3/2)\alpha_B^2\geq 0$, in both α_B and the kineticity α_K [14]. Moreover, g and α_B also enter in $G_{\rm eff}$, so even in the limit where the density and pressure are dominated by the kinetic term the theory and its observational implications lie outside k-essence.

V. DEVIATING THE KINETIC STRUCTURE

Moving away from the natural case where all ratios are of order one, suppose XK_X/K is not of order one. If it is very small, this is of no concern since then simply $\rho_{\rm de}\approx -K$ and $w\approx -1$. However large XK_X/K requires special attention. This is because while it enters in $\rho_{\rm de}$, only K enters in $P_{\rm de}$. If $\rho_{\rm de}/K$ becomes large enough, then the α_B term in the pressure could dominate over the first term.

Consider a Dirac-Born-Infeld (DBI) kinetic term (see e.g. [16, 17]). Then

$$K(X) = -\sqrt{1-X/X_\infty} \tag{16}$$

$$n \equiv \frac{XK_X}{K}=\frac{-X/X_\infty}{2(1-X/X_\infty)}\ , \tag{17}$$

and $n\to-\infty$ as $X\to X_\infty$. As seen by Eq. (15) this would drive $w\to 0^-$, or at least to $\mathcal{O}(\alpha_B)$. Thus an interesting scenario for the future of dark energy is that it could end up scaling as matter.

VI. DEVIATING THE FIELD EVOLUTION

Changing $\ddot\phi/(3H\dot\phi)$ from order one also has interesting consequences. Consider $\ddot\phi\gg 3H\dot\phi$. If the cosmic expansion rate H is the only timescale then this cannot be achieved, so such a condition requires introduction of a new time scale.

Recall that in the standard Klein-Gordon equation one has the acceleration term $\ddot\phi$, the friction term $3H\dot\phi$, and the driving term from the steepness of the potential, $-dV/d\phi$. As clearly illustrated in [18] for the quintessence case, none of these terms dominate over the others (unlike in slow roll inflation). Here we have no potential, but the driving term arises from $-6g(\dot H+3H^2)$.

One needs an extra ingredient entering the driving term, an additional, shorter, time scale than the expansion time scale H^{-1}. One example is a mass scale M, e.g.

$$g(X)=\gamma(X)+M\ . \tag{18}$$

However this appears problematic as a large $\ddot\phi$, increasing at faster than the Hubble rate, will eventually lead to a similarly large $\dot\phi$ and hence a large $\alpha_B\sim\dot\phi g/H$. That will cause large deviations from GR, with $G_{\rm eff}$ possibly diverging and instability from likely negative sound speed of scalar perturbations, $c_s^2<0$ (see [15]). So this case appears problematic.

VII. DEVIATING BY SLOW ROLL

We can take the opposite limit of field evolution: $\ddot\phi=0$, known as slow roll in inflation. Recall that we absolutely cannot do this for generic quintessence when the dark energy does not fully dominate [18], i.e. as in our present or past universe where there is matter. Taking $\ddot\phi=0$ would imply $\phi\sim t$, i.e. the only clock is the field, hence the field must dominate the cosmic expansion.

Suppose the $\ddot\phi=0$ condition holds, at some time at least. The field equation (5) then has the interesting property that it determines the action function g in terms of K (though only in this limit),

$$g=\frac{-\dot\phi K_X}{3H(1-w_{\rm dom})}\ , \tag{19}$$

where we have used $\dot H=(-3/2)(1+w_{\rm dom})H^2$, and $w_{\rm dom}$ indicates the equation of state of the dominant energy density component[2].

Keeping $\alpha_B\ll 1$ will imply $w\to-1$ when dark energy dominates, as

$$\alpha_B\equiv\frac{2\dot\phi g}{H}=\frac{-4XK_X}{3H^2(1-w)}\approx\frac{4n}{(1-2n)(1-w)}\ . \tag{20}$$

Thus $\alpha_B\ll 1$ requires $|n|\ll 1$, and hence by Eq. (15) that $w\to-1$. This implies that after dark energy crosses the phantom divide to $w>-1$, in this specific $\ddot\phi=0$ future it must turn around and go back to a de Sitter limit.

VIII. NO KINETIC TERM

We know that the g (i.e. G_3) term must exist, otherwise we are left with k-essence, which cannot cross the phantom divide by itself. Let us consider whether we can do without the K term, having only the G_3 term in the action.

When $K=0$, the field equation gains an interesting structure,

$$0 = \ddot\phi\, 6H\dot\phi g_X+6g\left(\dot H+3H^2\right) \tag{21}$$

$$= 6a^{-3}\left[(a^3H)\dot g+g\left(a^3H\right)^{\cdot}\right]\ , \tag{22}$$

since $\dot g=g_X\dot X=g_X\dot\phi\ddot\phi$. The solution is

$$g\sim\left(a^3H\right)^{-1}\ . \tag{23}$$

[2] This solution is quite similar to the Branch A solution of [19] that can well temper, i.e. cancel, a high energy cosmological constant, when we have a de Sitter state, with $w_{\rm dom}=-1$, $H\to h=$const. However canceling Λ only at late times is not so useful.